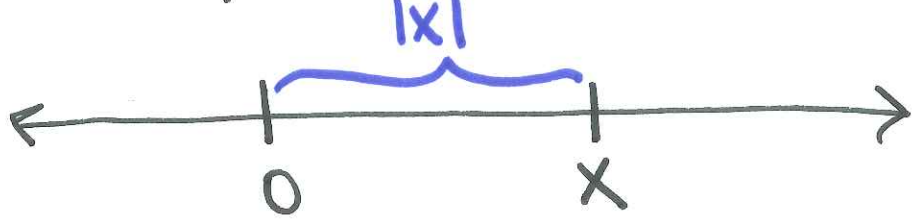
Chapter 3

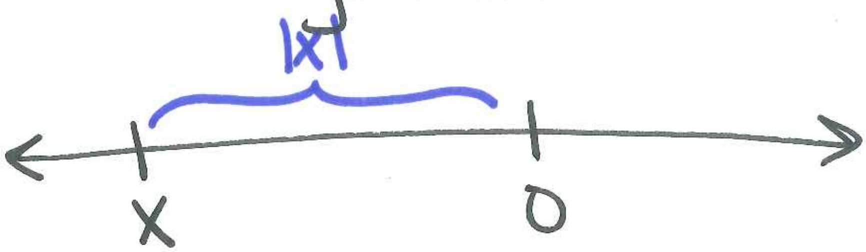
3.3

The <u>absolute value</u> of a number x, denoted $|x|$, is the distance between x and 0 on a number line.

if x is positive: $\qquad |x| = x$



if x is negative: $\qquad |x| = -x$



algebraically, we write

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Ex:

a) $|2.4| = \boxed{2.4}$

b) $|-\pi| = \boxed{\pi}$

c) $|4-\sqrt{2}| = \boxed{4-\sqrt{2}}$ $\sqrt{2} \approx 1.4$

d) $|\sqrt{2}-6| = -(\sqrt{2}-6)$

$= -\sqrt{2}+6$

$= \boxed{6-\sqrt{2}}$

Properties of Absolute Value

① $|c| \geq 0$

② if $c \neq 0$ then $|c| > 0$

③ $|c| = |-c|$

④ $|cd| = |c| \cdot |d|$

⑤ $\left|\frac{c}{d}\right| = \frac{|c|}{|d|}$

Pg 10 in textbook

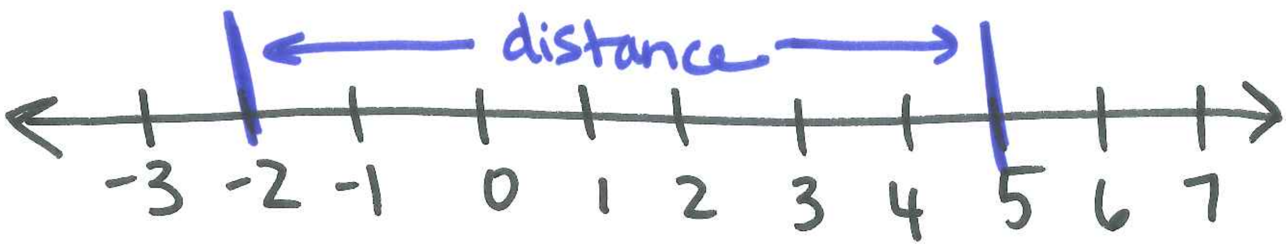
Special Properties

⑥ if c is a real number, then
$$\sqrt{c^2} = |c|$$

⑦ if x and y are real numbers, then $|x-y| = |y-x|$

We can also use absolute values to find the distance between two numbers

Ex: Find the distance between -2 and 5 on the number line.



$$|5-(-2)| = |7| = 7$$

OR $$|-2-5| = |-7| = 7$$

The distance between x and y on the number line is $|x-y|$

3.4

An <u>equation</u> is two expressions set equal to each other.

A <u>solution</u> to an equation is a value that makes the equation true.

<u>Ex</u>: Which is a solution to $3-5x=2(4-x)+1$?

a) $x=-2$

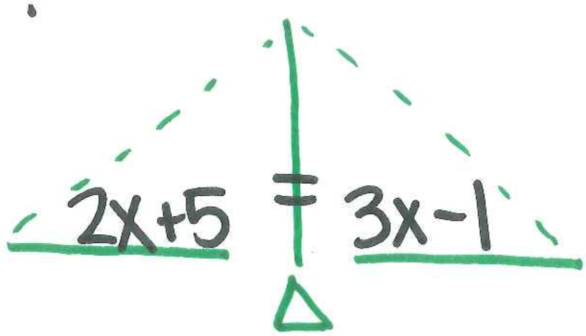
$$3-5(-2) \stackrel{?}{=} 2(4-{}^{-}2)+1$$
$$3+10 \stackrel{?}{=} 2(6)+1$$
$$13 \stackrel{?}{=} 13 \checkmark \quad \boxed{\text{yes - solution}}$$

b) $x=4$

$$3-5(4) \stackrel{?}{=} 2(4-4)+1$$
$$3-20 \stackrel{?}{=} 2(0)+1$$
$$-17 \stackrel{?}{=} 1$$
$$\neq \quad \boxed{\text{no - not a solution}}$$

3.4.1

Think of equations on a balance scale... what we do to one side, we must do to the other.



We want to use a series of <u>equivalent</u> equations to solve – this means that all the equations have the same solutions.

$$\left.\begin{aligned} 2x+5 &= 3x-1 \\ 5 &= x-1 \\ 6 &= x \end{aligned}\right\} \text{equivalent equations}$$

Operations that Produce Equivalent Equations:

1. Add/Subtract the same number/expression to each side.

2. Multiply/Divide both sides by a nonzero number.

3. Add zero to one side.

4. Multiply one side by 1.

Ex: Do the following produce equivalent equations?

a) Squaring both sides.

$\boxed{\text{Sometimes}}$

$x=0$
$x^2=0$
✓

$x=3$
$x^2=9$ → has 2 solutions $x=3,-3$
X

b) adding x to both sides

$\boxed{\text{always}}$ - see #1 above

c) Multiplying both sides by x

$\boxed{\text{Sometimes}}$

$x=0$ $x=3$

$x^2=0\cdot x$ $x^2=3x$ → has 2 solns $x=0,3$

✓

Sometimes we need to do these operations to solve, but they produce <u>extraneous solutions</u>. We must then

Check our Solutions!